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ON SOME FORMS OF LAGRANGE'S INTERPOLATION FORMULA.

By **Prof. W. H. Echols**, Charlottesville, Va.

1. The well known form

$$\frac{\begin{vmatrix} fx\ , & fa_1\ , & \ldots\ , & fa_n \\ 1\ , & 1\ , & \ldots\ , & 1 \\ x\ , & a_1\ , & \ldots\ , & a_n \\ \cdot & \cdot & \cdot & \cdot \\ x^{n-1}, & a_1^{\,n-1}, & \ldots\ , & a_n^{\,n-1} \end{vmatrix}}{\zeta^{\frac{1}{2}}(a_1 \ldots a_n)}\ , \tag{1}$$

which has for its value

$$R = (x - a_1) \ldots (x - a_n) \frac{f^n u}{n!}\ ,$$

(u lying between the greatest and least of x, a_1, ... a_n) may be written out according to the first row, in the standard form

$$fx = \sum_{r=1}^{r=n}{}' fa_r \frac{\Pi\,(x - a_i)}{\Pi\,(a_r - a_i)} + R\ , \quad (i = 1 \ldots n)\ \text{Ex}\ r \tag{2}$$

or by the first row or column according as we give precedence to the Σ sign of r or p, in the formula

$$fx = \sum_{r=1}^{n}{}' \sum_{p=0}^{n-1}{}' (-1)^{r+1} (-1)^{p+1} x^p fa_r P_p \frac{\Pi a_i}{\Pi\,(a_i - a_r)} + R\ , \tag{3}$$

wherein $P_0 = 1$, P_p is the sum of the products of the quantities $1/a_i$ taken p at a time, without repetition.

The values of

$$P_p \frac{\Pi a_i}{\Pi\,(a_i - a_r)}\ ,$$

for the three cases $a_r = r$, $a_r = r^2$, $a_r = a^r$, are, respectively,

$$\frac{n!}{r!\,(n - r)!} P_p\ , \tag{i}$$

$$2P_p \frac{(1 - 1/n) \ldots (1 - \overline{r - 1}/n)}{(1 + 1/n) \ldots (1 + r/n)}\ ; \tag{ii}$$

which becomes, when $n = \infty$,

$$2\,{}^r\mathfrak{F}_p\,,$$

${}^r\mathfrak{F}_p$ being Fourier's number of the rth order and class p.

$$\frac{P_p}{(a-1)\ldots(a^r-1)(1-1/a)\ldots(1-1/a^{n-r})}\,, \tag{iii}$$

in which

$$P_p = \sum_{q=0}^{p} \frac{(-1)^q}{a^{rq}(a-1)\ldots(a^{p-q}-1)}$$

$$= S_p - \frac{1}{a^r}P_{p-1}\,,$$

$$S_p \quad (a-1)\ldots(a^p-1)\,.$$

The corresponding values of R are in these cases

$$R = (-1)^n(1-x/1)\ldots(1-x/n)\,f^n u\,, \tag{i}$$

$$R = (-1)^n(1-x/1^2)\ldots(1-x/n^2)\,n!\,f^n u\,, \tag{ii}$$

$$R = (-1)^n(1-x/a)\ldots(1-x/a^n)\,a^{\frac{1}{2}n(n+1)}f^n u/n! \tag{iii}$$

2. Instead of writing the determinant (1) in solid columns, we may write it

$$\frac{\begin{vmatrix} fx & , x^{n|1} & , x^{n|2} & , \ldots , & x^{n|n} \\ fa_0 & , a_0^{n|1} & , a_0^{n|2} & , \ldots , & a_0^{n|n} \\ fa_1 & , 0 & , a_1^{n|2} & , \ldots , & a_1^{n|n} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ fa_{n-1} & , 0 & , 0 & , \ldots , & a_{n-1}^{n|n} \end{vmatrix}}{a_0^{n|1}\ldots a_{n-1}^{n|n}}\,, \tag{4}$$

wherein

$$x^{n|r} = (x-a_n)\ldots(x-a_r)\,.$$

(4) gives

$$fx = \sum_{p=0}^{n-1} A_p fa_p + \frac{x^{n|0}}{(n+1)!}f^n u\,. \tag{5}$$

in which we have for computing the A's, $A_0 = x^{n|1}/a_0^{n|1}$, and

$$(-1)^p a_p^{n|(p+1)}A_p = a_{p-1}^{n|(p+1)}A_{p-1} - a_{p-2}^{n|(p+1)}A_{p-2} + \ldots + (-1)^{p-1}a_0^{n|(p+1)}A_0$$
$$+ (-1)^p x^{n|(p+1)}\,.$$

3. Again, we have

$$\frac{\begin{vmatrix} fx, & x^{1|0}, & x^{2|0}, & \dots, & x^{(n-1)|0} \\ fa_1, & a_1^{1|0}, & 0, & \dots, & 0 \\ fa_2, & a_2^{1|0}, & a_2^{2|0}, & \dots, & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ fa_{n-1}, & a_{n-1}^{1|0}, & a_{n-1}^{2|0}, & \dots, & a_{n-1}^{(n-1)|0} \end{vmatrix}}{a_1^{1|0} \dots a_{n-1}^{(n-1)|0}}, \tag{6}$$

wherein

$$x^{n|0} = (x - a_n) \dots (x - a_0) .$$

(6) gives

$$fx = \sum_{r=1}^{n-1} A_r x^{r|0} + \frac{x^{n|0}}{n!} f^n u , \tag{7}$$

in which we have for computing the A's, $A_1 = fa_1/a_1^{1|0}$, and

$$(-1)^{r+1} a_r^{(r-1)|0} A_r = a_r^{(r-1)|0} A_{r-1} - a_r^{(r-2)|0} A_{r-2} + \dots + (-1)^r a_r^{1|0} A_1 \\ + (-1)^{r+1} fa_r .$$

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